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Laue soliton in a resonantly absorbing photonic crystal

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Abstract

A theory has been developed for coherent nonlinear light–matter interaction under condition of Bragg diffraction in a multidimensional resonantly absorbing photonic crystal. At the Laue scheme of diffraction in a two-wave approximation, a novel kind of the nonlinear solitary wave, the so-called Laue soliton of self-induced transparency, has been obtained as an analytical and numerical solution of corresponding Maxwell–Bloch equations. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In the last decade, much attention has been paid to study of light–matter interaction in photonic crystals (PC) [1]. PC are one-, two-, or three-dimensional structures with a periodically modulated dielectric function and, moreover, the period of modulation is close to optical wavelength. If light–matter interaction is linear and the contrast of the dielectric function is low, light propagation in PC is similar to X-ray diffraction in a traditional crystal [2]. The high contrast of PC dielectric function leads to a complete photonic band gap, where light propagation is inhibited [3,4]. Another specific distinction of electromagnetic field dynamics in PC is displayed when a nonlinearity of the interaction is taken into account [5–10]. For the resonantly absorbing discrete PC, that is formed by the periodically distributed thin layers of two-level atoms, this is the existence of nonlinear solitary waves, or gap solitons of self-induced transparency [7–9]. These waves propagate through the structure at Bragg frequency within the linear forbidden gap band of the periodic medium. The gap soliton is formed by two counterpropagating coupled Bloch modes at the reflection geometric scheme of Bragg diffraction in one-dimensional (1D) PC. However, it is well known another scheme of Bragg diffraction, the so-called Laue transmission scheme [2]. In this case an incoming field falls on a structure boundary oriented along reciprocal lattice vector, then the coupled diffracted waves can propagate through the structure without strong reflection at the boundary, because there is no photonic band gap in direction of

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normal to the reciprocal lattice vector. The linear field dynamics in multidimensional structure under condition of the Laue diffraction, for instance the Borrmann effect, has been investigated in X-ray optics in detail [2]. The progress in technology allows now to fabricate multidimensional PC [1]. In contrast with the previous studies of both the linear Laue diffraction [2] and the nonlinear pulse propagation under the condition of Bragg reflection in 1D PC [5–10], this paper will focus on the problem of nonlinear Bragg diffraction in multidimensional PC, especially at the Laue scheme of diffraction.

In the present paper, we study the propagation of field in resonantly absorbing discrete multidimensional PC. The equations of two-wave nonlinear dynamical Bragg diffraction for non-counterpropagating waves have been derived from the semiclassical Maxwell–Bloch equations. They describe the coherent light–matter interaction in discrete PC both for the Bragg reflection and for the Laue transmission two-dimensional (2D) geometric schemes of Bragg diffraction. It has been shown, as we believe for the first time, that nonlinear diffraction of resonant field at the Laue scheme gives rise to formation of four diffracted modes within the PC. Two waves form the so-called slow Laue soliton of self-induced transparency, while two other modes are coupled in a fast linear propagating field. The nodes of the linear propagating waves are located at crystallographic planes of the PC containing the resonant two-level atoms. As a result, all four diffracted waves are able to propagate through the resonantly absorbing structure without absorption.

2. Equations of dynamical nonlinear two-wave Bragg diffraction in a resonantly absorbing photonic crystals

The periodically distributed small domains doped with resonant two-level atoms in uniform host medium form a three-dimensional (3D) space lattice in our model of resonantly absorbing discrete PC. The density of resonant atoms must be low enough, in order to neglect a modulation of the linear dielectric constant of the medium, as well a dipole–dipole interaction of atoms. The size of each domain near the lattice point is assumed to be small as compared with the wavelength λ, and the lattice spacing d must be comparable to λ.

Maxwell's equations lead to the following wave equation for electric field $\vec{E}(\vec{r},t)$ of light within the structure:

$$\mathrm{rotrot}\,\vec{E}(\vec{r},t) + \frac{\varepsilon}{c_0^2}\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2} = -\frac{4\pi}{c_0^2}\frac{\partial^2\vec{P}(\vec{r},t)}{\partial t^2}, \qquad (1)$$

where ε is the dielectric constant of the medium, the linear dispersion is neglected, c_0 is the light velocity in vacuum, the nonlinear polarization due to the two-level atoms is

$$\vec{P}(\vec{r},t) = \sum_j N\vec{P}'_j(t)\,\delta(\vec{r}-\vec{r}_j). \qquad (2)$$

Here vector $\vec{r}_j$ defines the location of the jth lattice point, N is the number of two-level atoms in a domain, $\delta(\vec{r}-\vec{r}_j)$ is the Dirac delta function, $\vec{P}'_j = (\vec{P}_j/2)\exp(-\mathrm{i}\omega t) + \mathrm{c.c.}$ is single-atom polarization, $\vec{P}_j(t)$ is slowly varying in time complex atomic polarization which will be obtained from Bloch equations, and ω is the frequency of light.

The reciprocal lattice of the PC is 3D. However, if two diffracted wave vectors $\vec{k}_{0,h}$ and the reciprocal lattice vector $\vec{H}$ exactly satisfy the Bragg condition $\vec{k}_h = \vec{k}_0 + \vec{H}$, we are able to replace a 3D problem of diffraction by a 2D problem using two-wave approximation. Under this assumption, the quasimonochromatic field $E(\vec{r},t)$ within the structure can be written as a sum of two strong Bloch modes

$$E(\vec{r},t) = (1/2)[E_0(\vec{r},t)\exp(\mathrm{i}\vec{k}_0\vec{r} - \mathrm{i}\omega t) + E_h(\vec{r},t)\exp(\mathrm{i}\vec{k}_h\vec{r} - \mathrm{i}\omega t)] + \mathrm{c.c.} \qquad (3)$$

Here E_0 and E_h are slowly varying in time $|\partial E_{0,h}/\partial t| \ll \omega|E_{0,h}|$ and in space $|\partial E_{0,h}/\partial\vec{r}| \ll k_{0,h}|E_{0,h}|$ envelopes of complex amplitudes of the incident and diffracted waves. The modulus of reciprocal lattice vector $|\vec{H}| = 2\pi/d$.

If the linear polarized electric field is always perpendicular to the directions of vectors $\vec{k}_0$ and $\vec{k}_h$ (s-polarization), Eq. (1) takes the form

$$\frac{\partial^2 E}{\partial\vec{r}^2} - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2}. \qquad (4)$$

Using the slowly varying envelope approximation [11], substituting Eqs. (2) and (3) into Eq. (4) and averaging over a time $\Delta t \gg \omega^{-1}$ and a volume $V \gg \lambda^3$, we obtain the following two equations for the wave amplitudes:

$$\begin{aligned} &\frac{\partial E_0}{\partial \vec{r}}\vec{k}_0 + \frac{k^2}{\omega}\frac{\partial E_0}{\partial t} = \frac{2\pi \mathrm{i} N k^2}{\varepsilon} \\ &\quad \times \left\langle \sum_j P_j(t)\exp(-\mathrm{i}\vec{k}_0\vec{r})\,\delta(\vec{r}-\vec{r}_j)\right\rangle_V, \\ &\frac{\partial E_h}{\partial \vec{r}}\vec{k}_h + \frac{k^2}{\omega}\frac{\partial E_h}{\partial t} = \frac{2\pi \mathrm{i} N k^2}{\varepsilon} \\ &\quad \times \left\langle \sum_j P_j(t)\exp(-\mathrm{i}\vec{k}_h\vec{r})\,\delta(\vec{r}-\vec{r}_j)\right\rangle_V, \end{aligned} \tag{5}$$

where the angular brackets denote the averaging over a volume V, and $k \equiv |\vec{k}_0| = |\vec{k}_h|$. It is assumed that averaged oscillating terms are small

$$\left|\left\langle \frac{\partial E_{0,h}}{\partial \vec{r}}\vec{k}_{0,h}\exp(\pm \mathrm{i}\vec{H}\vec{r})\right\rangle_V\right| \ll \left|\frac{\partial E_{0,h}}{\partial \vec{r}}\vec{k}_{0,h}\right|$$

and may be neglected. Due to the Bragg condition, the function $\exp(\mathrm{i}\vec{H}\vec{r}_j) = 1$ in a discrete model of PC, and

$$\begin{aligned} \exp(-\mathrm{i}\vec{k}_h\vec{r}_j) &= \exp(-\mathrm{i}\vec{k}_0\vec{r}_j - \mathrm{i}\vec{H}\vec{r}_j) \\ &= \exp(-\mathrm{i}\vec{k}_0\vec{r}_j). \end{aligned} \tag{6}$$

Then, the right-hand parts of Eq. (5) are equal and can be expressed through the slowly varying complex amplitude of polarization of the structure P_s

$$\begin{aligned} P_s(\vec{r},t) &\equiv N\left\langle \sum_j P_j(t)\exp(-\mathrm{i}\vec{k}_0\vec{r})\,\delta(\vec{r}-\vec{r}_j)\right\rangle_V \\ &= N\left\langle \sum_j P_j(t)\exp(-\mathrm{i}\vec{k}_h\vec{r})\,\delta(\vec{r}-\vec{r}_j)\right\rangle_V. \end{aligned} \tag{7}$$

The dynamics of coherent interaction of two-level atoms with the resonant field is described by optical Bloch equations for complex atomic polarization P_j and the population difference of the atoms n_j [11]. These equations can be obtained, for instance, from the equation of motion for the density matrix of a two-level system in superposition state. In the case of exact frequency resonance, the Bloch equations for atoms in the field (3) near the jth lattice point take the following form [10]:

$$\begin{aligned} \frac{\mathrm{d}P_j}{\mathrm{d}t} &= -\frac{\mathrm{i}}{\hbar}\mu^2[E_0(\vec{r}_j,t)\exp(\mathrm{i}\vec{k}_0\vec{r}_j) \\ &\qquad + E_h(\vec{r}_j,t)\exp(\mathrm{i}\vec{k}_h\vec{r}_j)]n_j, \\ \frac{\mathrm{d}n_j}{\mathrm{d}t} &= \frac{2\mathrm{i}}{\hbar}\{[E_0(\vec{r}_j,t)\exp(\mathrm{i}\vec{k}_0\vec{r}_j) \\ &\qquad + E_h(\vec{r}_j,t)\exp(\mathrm{i}\vec{k}_h\vec{r}_j)]P_j^* - \mathrm{c.c.}\}, \end{aligned} \tag{8}$$

where $P_j = \mu\rho'_{21}$, ρ'_{21} is the slowly amplitude of the function $\rho_{21} = \rho'_{21}\exp(\mathrm{i}\omega t)$, $n_j = \rho_{22} - \rho_{11}$, ρ_{il} are the matrix elements of density matrix, μ is the matrix element of the transition dipole moment.

After we average Eq. (8) over a region V, taking into account the discrete distribution of the density of resonant atoms in PC, as well the expressions (6) and (7), the self-consistent set of Maxwell–Bloch equations (5) and (8) takes the form

$$\begin{aligned} &\frac{\partial E_0}{\partial \vec{r}}\vec{k}_0 + \frac{k^2}{\omega}\frac{\partial E_0}{\partial t} = \frac{2\pi \mathrm{i} k^2}{\varepsilon}P_s, \\ &\frac{\partial E_h}{\partial \vec{r}}\vec{k}_h + \frac{k^2}{\omega}\frac{\partial E_h}{\partial t} = \frac{2\pi \mathrm{i} k^2}{\varepsilon}P_s, \\ &\frac{\mathrm{d}P_s}{\mathrm{d}t} = -\frac{\mathrm{i}}{\hbar}\mu^2(E_0+E_h)\rho, \\ &\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{2\mathrm{i}}{\hbar}\left[(E_0+E_h)P_s^* - (E_0+E_h)^*P_s\right]. \end{aligned} \tag{9}$$

Here $\rho(r,t)$ is the density of atomic inverse population. Finally, it is convenient for further analysis to introduce the parameter of coherent interaction τ_c characterizing the mean photon lifetime in the medium preceding resonant absorption [7–9]. Rewriting Eq. (9) for the functions $R = 2\mathrm{i}P_s/\mu\sigma$, $n = \rho/\sigma$, and $\Omega_{0,h} = 2(\mu/\hbar)E_{0,h}$, one obtains

$$\begin{aligned} &c\frac{\partial\Omega_0(\vec{r},t)}{\partial\vec{k}_0} + \frac{\partial\Omega_0(\vec{r},t)}{\partial t} = \tau_c^{-2}R(\vec{r},t), \\ &c\frac{\partial\Omega_h(\vec{r},t)}{\partial\vec{k}_h} + \frac{\partial\Omega_h(\vec{r},t)}{\partial t} = \tau_c^{-2}R(\vec{r},t), \\ &\frac{\partial R(\vec{r},t)}{\partial t} = n(\vec{r},t)[\Omega_0(\vec{r},t) + \Omega_h(\vec{r},t)], \\ &\frac{\partial n(\vec{r},t)}{\partial t} = -\mathrm{Re}\{R^*(\vec{r},t)[\Omega_0(\vec{r},t) + \Omega_h(\vec{r},t)]\}, \end{aligned} \tag{10}$$

where the directional derivative is given by $\partial\Omega/\partial\vec{k} \equiv (\partial\Omega/\partial\vec{r})(\vec{k}/k)$, the cooperative time is given by the expression $\tau_c^2 = 8\pi\varepsilon T_1/3c_0\sigma\lambda_0^2$, $\lambda_0 = 2\pi c_0/\omega$,

$c = c_0/(\varepsilon)^{1/2}$, the atomic exited level lifetime $T_1 = 3\hbar c_0^3/4\omega^3\mu^2$, and σ is the average density of resonant atoms. The Bloch equations in the form (10) describe the rotation of unit Bloch vector $\{\mathrm{Re}\, R, \mathrm{Im}\, R, n\}$ at the angular velocity $\Omega = \Omega_0 + \Omega_h$ [11].

Eq. (10) describe the spatial-temporal dynamics of the field and atomic subsystem under condition of nonlinear two-wave Bragg diffraction in multidimensional resonantly absorbing PC. Note, that Eq. (10) differ from 1D equations for counter-propagating Bloch waves [7–9]. They allow us to solve more general problems of nonlinear diffraction using different 2D schemes of Bragg diffraction: the Bragg reflection and the Laue transmission schemes.

3. Laue soliton of self-induced transparency

Let us consider the nonlinear diffraction at the Laue scheme. Fig. 1 illustrates the orientation of wave vectors and reciprocal lattice vector in this case. The diffracted waves are coupled due to the reflection on crystallographic planes within the structure. Let the symmetrical diffraction scheme be realized $\varphi = \psi$ (Fig. 1), and fields are homogeneous with respect to the y-coordinate $\partial\Omega_{0,h}/\partial y = 0$. Then Eq. (10) take the form

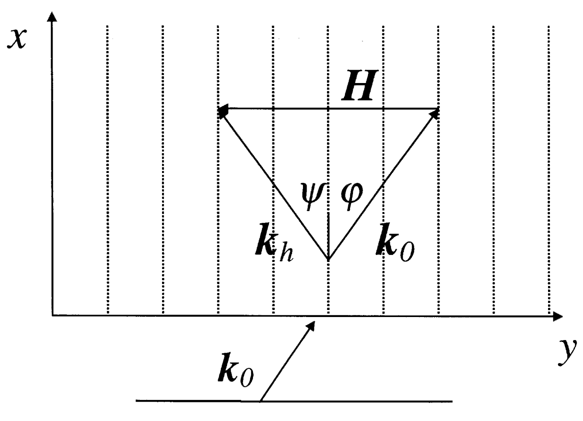

Fig. 1. The Laue scheme of diffraction on crystallographic planes of PC. The angles of diffraction φ and ψ are the angles between wave vectors and x-axis.

$$
\begin{aligned}
&c\cos\varphi\frac{\partial\Omega_0}{\partial x} + \frac{\partial\Omega_0}{\partial t} = \tau_c^{-2}R,\\
&c\cos\varphi\frac{\partial\Omega_h}{\partial x} + \frac{\partial\Omega_h}{\partial t} = \tau_c^{-2}R,\\
&R = -\sin\theta, \qquad n = -\cos\theta,
\end{aligned}
\tag{11}
$$

where $\theta(x,t) = \int_{-\infty}^{t} \Omega_0(x,t') + \Omega_h(x,t')\,\mathrm{d}t'$ is the Bloch angle. It is not hard to transform from Eq. (11) to equation for the angle $\theta(x,t)$

$$
c\cos\varphi\frac{\partial^2\theta}{\partial x\,\partial t} + \frac{\partial^2\theta}{\partial t^2} = -2\tau_c^{-2}\sin\theta. \tag{12}
$$

Eq. (12) is the sine-Gordon equation in the form like that for the self-induced transparency problem in homogeneous medium [12]. Corresponding one-soliton solution of Eq. (12) yields

$$
\Omega(x,t) = \Omega_0 + \Omega_h = 2\tau_p^{-1}\mathrm{sech}[(t - x/v_p)/\tau_p], \tag{13}
$$

where pulse velocity is fixed by the expression $v_p = c\cos\varphi/(1 + 2\tau_p^2/\tau_c^2)$, τ_p is the pulse duration.

The slowly propagating soliton (13) consists of two diffracted waves. Since we are interested in the solutions for each mode Ω_0 and Ω_h, Eq. (11) must be transformed for the function $\Omega^- = \Omega_0 - \Omega_h$. Then one obtains the following linear equation

$$
c\cos\varphi\frac{\partial\Omega^-}{\partial x} + \frac{\partial\Omega^-}{\partial t} = 0.
$$

The solution of this equation is a linear wave

$$
\Omega^- = \Omega^-(\xi), \qquad \xi = x - (c\cos\varphi)t, \tag{14}
$$

which propagates within the structure at the fast velocity $c\cos\varphi$.

This result looks surprising, because the sum of two diffracted modes Ω (13) moves as a soliton at the slow velocity v_p, but the difference Ω^- (14) has the fast velocity $c\cos\varphi$. This is possible only in the cases if Ω or Ω^- is equal to zero.

Let us first consider the case, when the field sum is not equal to zero but the difference is zero: $\Omega_0 - \Omega_h = 0$, $\Omega_0 + \Omega_h = \partial\theta/\partial t \neq 0$. It means that amplitudes of two modes are equal. Using formula (13), we obtain the following solutions for both waves and atomic inverse population:

$$
\begin{aligned}
&\Omega_0 = \Omega_h = \Omega/2 = \tau_p^{-1}\mathrm{sech}[(t - x/v_p)/\tau_p],\\
&n = -1 + 2\,\mathrm{sech}^2[(t - x/v_p)/\tau_p].
\end{aligned}
\tag{15}
$$

Expressions (15) describe the soliton of self-induced transparency coupling two diffracted modes with equal amplitudes. It propagates in the direction of the normal to reciprocal lattice vector under condition of the Laue diffraction. We call them "Laue soliton". The Laue soliton differs from gap soliton [7–9] on principle, first of all because it is not "gap" soliton and moves in direction where there is no any photonic band gap. The gap soliton propagates along the reciprocal lattice vector, has other velocity and duration, and is formed by two Bloch modes with opposite signs of the amplitudes. Moreover, the Laue soliton consists of two noncounterpropagating waves and cannot be found in 1D problem of diffraction studied before [5–10].

Another case is realized when the field sum is zero but the difference is not equal to zero: $\Omega_0 + \Omega_h = 0$, $\theta = 0$, $\Omega_0 - \Omega_h \neq 0$. Hence

$$\Omega_0(\xi) = -\Omega_h(\xi), \quad n(\xi) = -1. \tag{16}$$

The linear diffracted modes of the field (16) propagate through the structure without interaction with resonant atoms ($n = -1$) even if the amplitude of each mode is large. In comparison with nonlinear modes of the Laue soliton (15), the linear field (16) couples two waves with opposite signs of the amplitudes.

To make clear the spatio-temporal dynamics of the Laue soliton and the linear diffracted modes in the structure, and to show that they can be produced under irradiation of the medium boundary by an incident field, we simulated numerically the process of light–matter interaction under condition of the Laue diffraction. The numerical integration of Eq. (10) uses the difference scheme. Fig. 2 illustrates the result of the simulation of nonlinear Laue diffraction of the incident pulse $\Omega_0(x = 0; y, t) = \Omega_0'(y)\mathrm{sech}[(t - t_0)/\tau_0]$ in the finite PC when the maximum of pulse amplitude $\Omega_0' = 2 \times 10^{13}\ \mathrm{s}^{-1}$, the pulse duration $\tau_0 = 0.3\tau_c$, $\tau_c = 3 \times 10^{-13}$ s, $\lambda = 500$ nm, and $\varphi = 30°$. The diffraction of this pulse gives rise to four diffracted waves within the structure (Fig. 2a and b). Two waves having identical envelope of the amplitudes form the Laue soliton. Another pear of coupled modes is characterized by opposite signs of amplitudes and forms the linear propagating field. The fast linear field outstrips the slow Laue soliton and does not excite the resonant atoms, while the motion of the Laue soliton is accompanied by the strong excitation of two-level atomic subsystem (Fig. 2c). This is because the total angular velocity

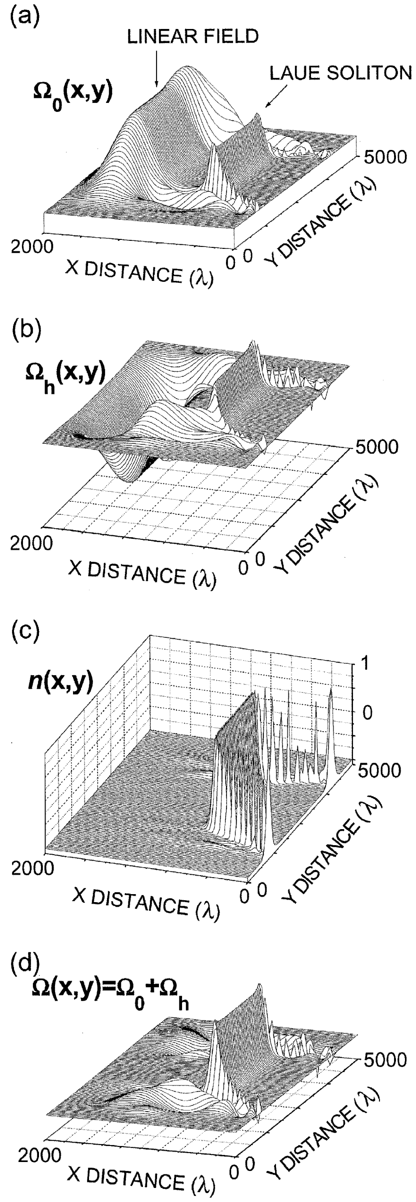

Fig. 2. Spatial distribution of the diffracted mode amplitudes Ω_0 (a) and Ω_h (b), the inverse population of resonant atoms n (c), as well the total angular velocity of Bloch vector rotation Ω (d) at the Laue diffraction of coherent light pulse in PC.

of Bloch vector rotation $\Omega(x,y,t) = \Omega_0 + \Omega_h$ is equal to zero within the area of coupled linear modes and has a large value inside the Laue soliton (Fig. 2d). The pulses velocity and duration, as well the value of diffracted mode amplitudes $\Omega_{0,h}$ agree with analytical expressions (15) and (16) obtained above.

4. Conclusion

The nonlinear theory of light diffraction in PC, developed here, is a nonlinear extension of the linear theory, which is used traditionally for diffraction problems of X-ray optics. In case of linear light–matter interaction at the Laue scheme of diffraction, four diffracted modes are emitted by two centers of propagation on dispersive curves [2]. Two waves with opposite signs of amplitudes propagate through the crystal feeling weak absorption because the nodes of total field are located at the crystallographic planes of the structure, the so-called Borrmann effect [2]. Other two waves are absorbed. These are the waves that form the Laue soliton of self-induced transparency at the nonlinear diffraction. Thus, due to the nonlinearity of the interaction, all four waves propagate within PC without absorption. Our results can be interpreted as the nonlinear Borrmann effect in resonantly absorbing PC.

The Laue soliton could be observed experimentally, for instance, in colloidal photonic crystals [13,14] or 2D structures of glass rods with embedded dye molecules [15], as well in a structure of air-rods filled with dye solution [16].

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