Восьмая всероссийская летняя школа учителей физики

2018

ПОПУРРИ НА ТЕМЫ НЕЛИНЕЙНОЙ ФИЗИКИ

Демонстрация незамысловатых механических систем, которые легко сделать своими руками

Кротов С.С., Шутеев С.А.

sskrotov@mail.ru, gsrg.shu@gmail.com

«Я не могу полностью принять новую идею, пока не найду ее простейшую механическую реализацию...»

Лорд Кельвин

«Многие вещи нам непонятны **не** потому, что наши **понятия** слабы; но потому, что сии вещи **не входят в круг наших понятий**»

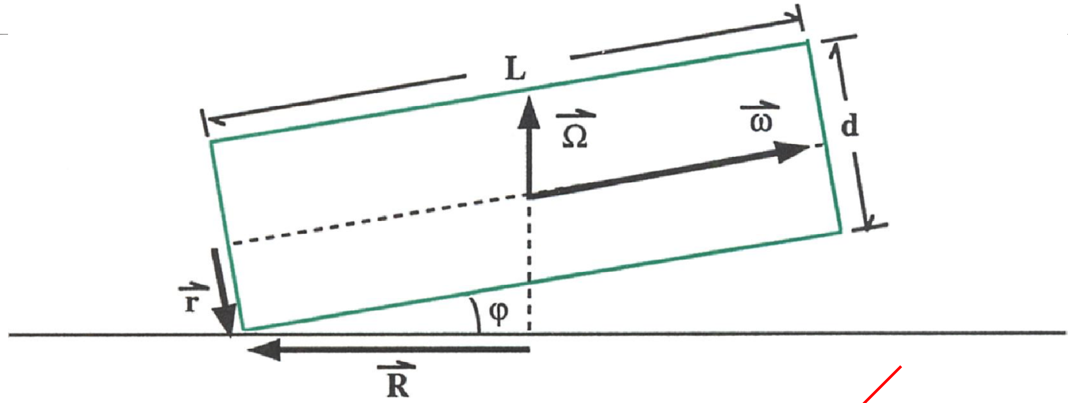
Козьма Прутков

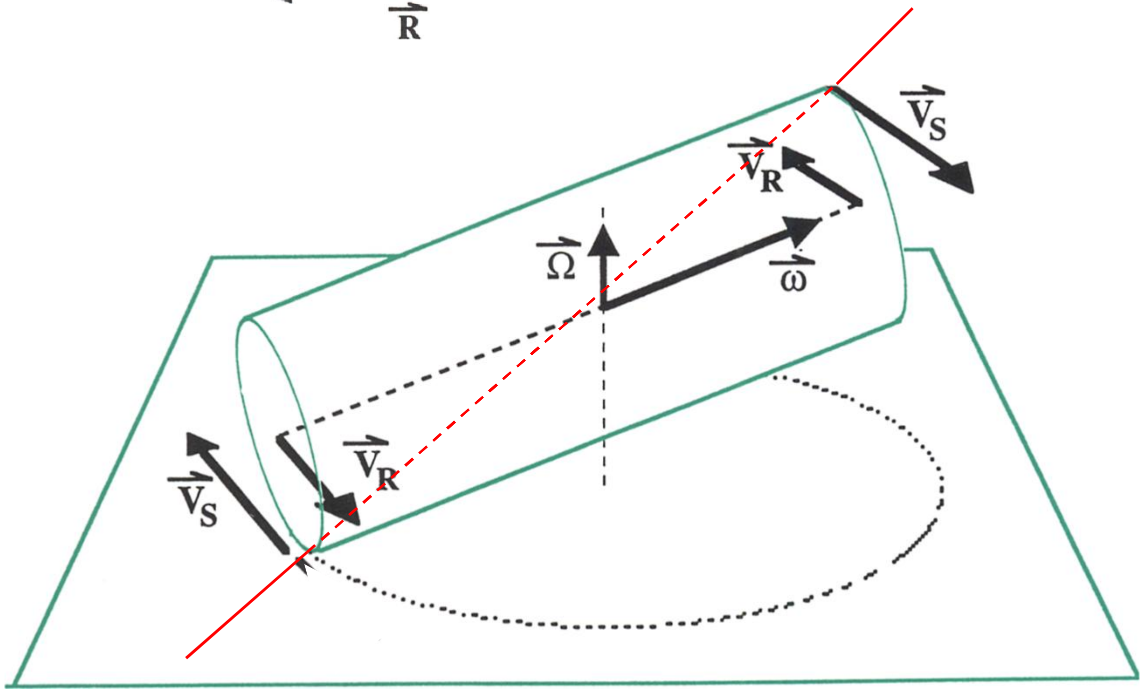
«Лучше один раз увидеть…»

Часть пословицы

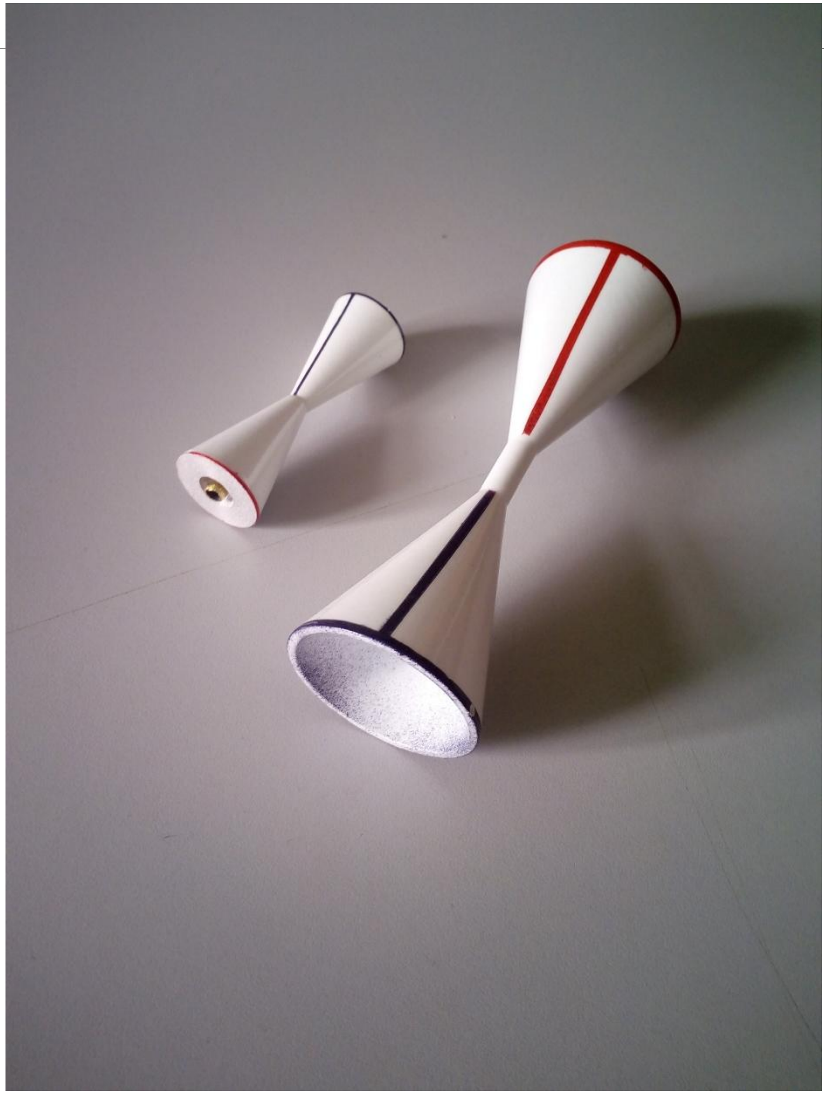
Вращающиеся трубки

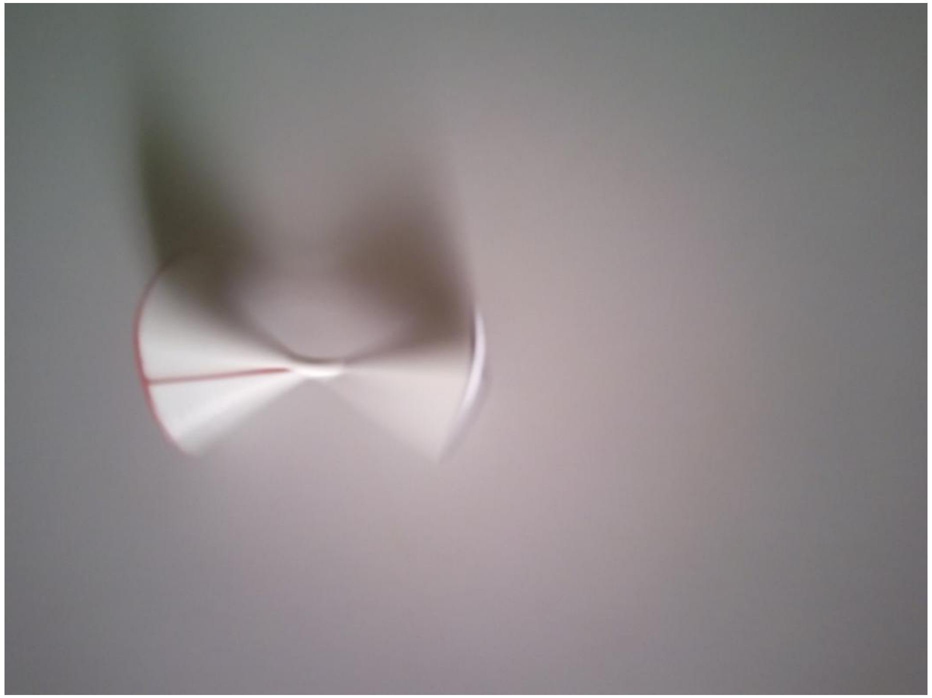
Вращающиеся трубки



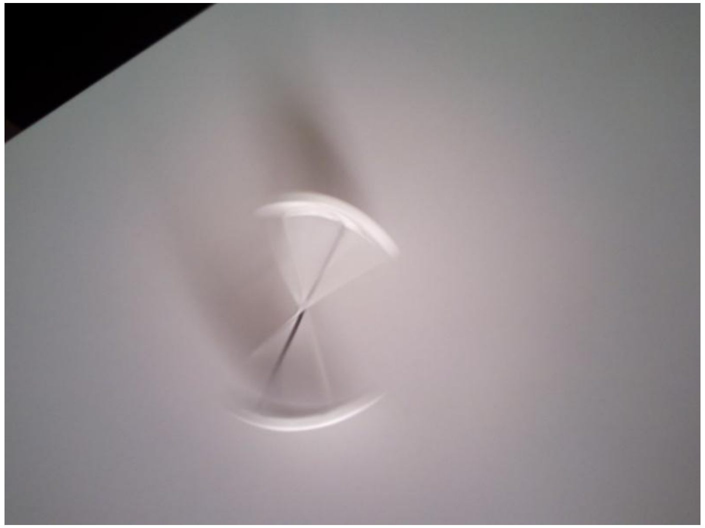


Вращающиеся конусы





Другие варианты



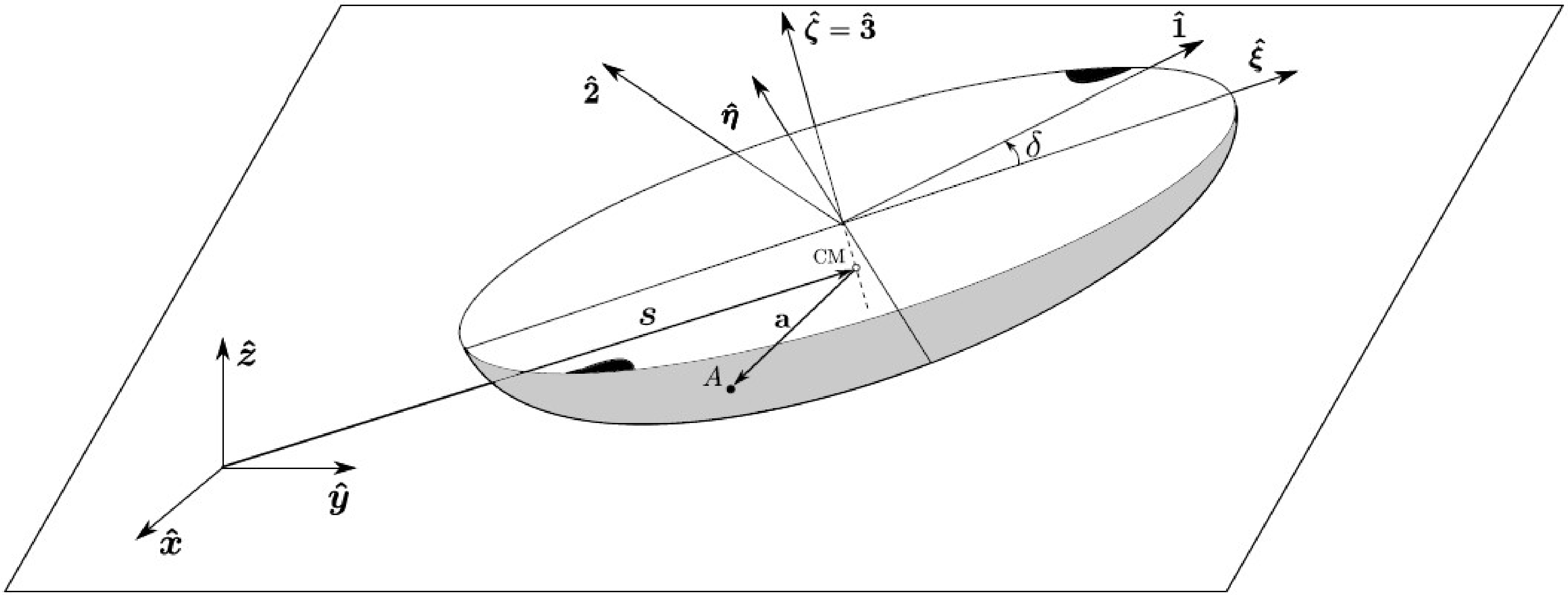
Другие варианты

Другие варианты

Удивительный Celt – «Кельтский камень»

Удивительный Celt – «Кельтский камень»

Кельтский камень называют также «anagyre», «rattleback», «celtic stone», «rebellious celt», «rattlerock», «spin bar», «wobble stone» или «wobblestone». Кроме того, имеются коммерческие названия: «ARK», «Bizzaro Swirls», «Space Pet» и «Space Toy».



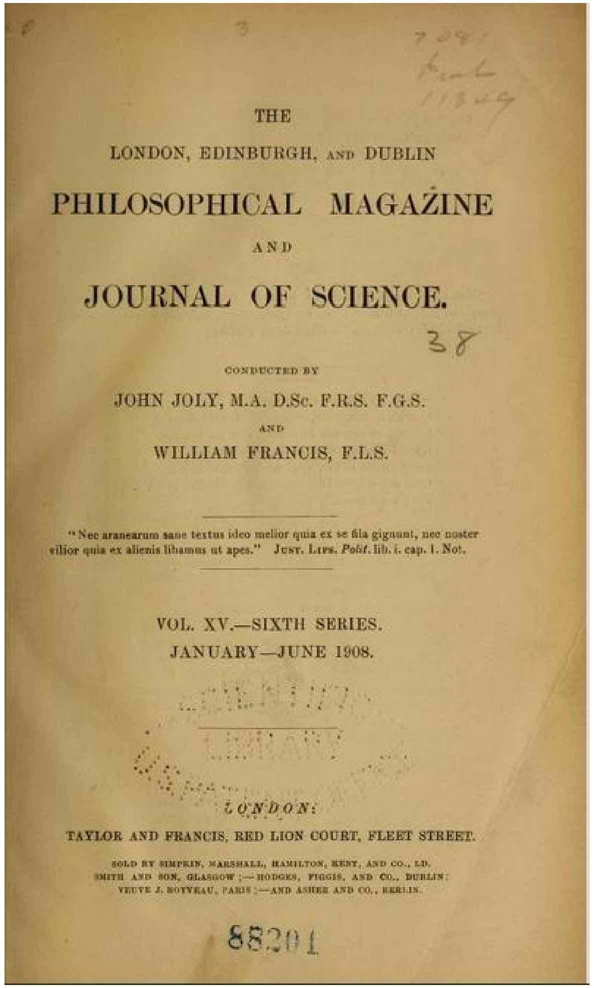
Кельтский камень, который совсем не камень

Маятник Капицы

История вопроса

Andrew Stephenson

1908г.“On an induced stability”*Phil. Mag*. **15**, 233 – 236



THE

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JOURNAL OF SCIENCE.

38

CONDUCTED BY

JOHN JOLY, M.A. D.Sc. F.R.S. F.G.S.

AND

WILLIAM FRANCIS, F.L.S.

“Nec aranearum sane textus ideo melior quia ex se fila gignunt, nec noster vilior quia ex alienis libamus ut apes.” JUST. LIPS. *Polit.* lib. i. cap. 1. Not.

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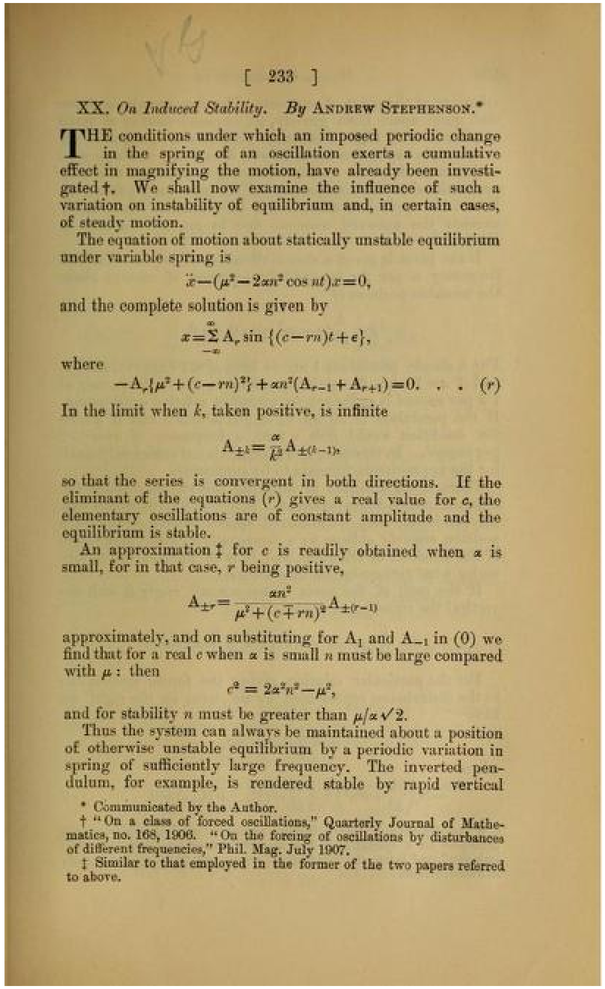


[233]

XX. *On Induced Stability.* *By* ANDREW STEPHENSON.*

THE conditions under which an imposed periodic change in the spring of an oscillation exerts a cumulative effect in magnifying the motion, have already been investigated†. We shall now examine the influence of such a variation on instability of equilibrium and, in certain cases, of steady motion.

The equation of motion about statically unstable equilibrium under variable spring is

$$\ddot{x}-(\mu^2-2\alpha n^2\cos nt)x=0,$$

and the complete solution is given by

$$x=\sum_{-\infty}^{\infty}A_r\sin\{(c-rn)t+\epsilon\},$$

where

$$-A_r\{\mu^2+(c-rn)^2\}+\alpha n^2(A_{r-1}+A_{r+1})=0. \quad . \quad . \quad (r)$$

In the limit when k, taken positive, is infinite

$$A_{\pm k}=\frac{\alpha}{k^2}A_{\pm(k-1)},$$

so that the series is convergent in both directions. If the eliminant of the equations (r) gives a real value for c, the elementary oscillations are of constant amplitude and the equilibrium is stable.

An approximation ‡ for c is readily obtained when α is small, for in that case, r being positive,

$$A_{\pm r}=\frac{\alpha n^2}{\mu^2+(c\mp rn)^2}A_{\pm(r-1)}$$

approximately, and on substituting for A_1 and A_{-1} in (0) we find that for a real c when α is small n must be large compared with μ : then

$$c^2=2\alpha^2n^2-\mu^2,$$

and for stability n must be greater than $\mu/\alpha\sqrt{2}$.

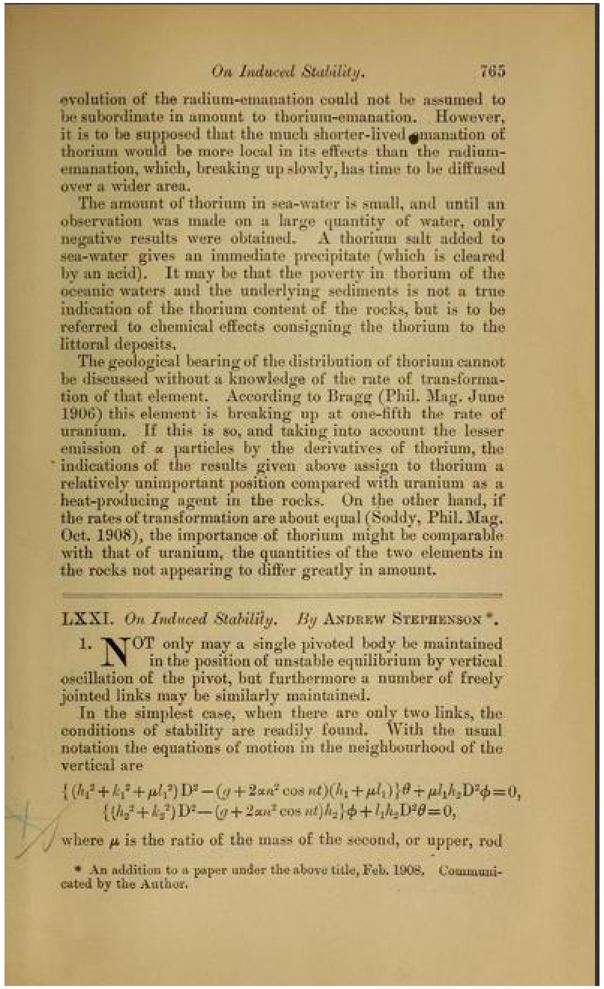
Thus the system can always be maintained about a position of otherwise unstable equilibrium by a periodic variation in spring of sufficiently large frequency. The inverted pendulum, for example, is rendered stable by rapid vertical

* Communicated by the Author.

† “On a class of forced oscillations,” Quarterly Journal of Mathematics, no. 168, 1906. “On the forcing of oscillations by disturbances of different frequencies,” Phil. Mag. July 1907.

‡ Similar to that employed in the former of the two papers referred to above.

1909г. *Phil. Mag*. **17**, 765 – 766



On Induced Stability. 765

evolution of the radium-emanation could not be assumed to be subordinate in amount to thorium-emanation. However, it is to be supposed that the much shorter-lived emanation of thorium would be more local in its effects than the radium-emanation, which, breaking up slowly, has time to be diffused over a wider area.

The amount of thorium in sea-water is small, and until an observation was made on a large quantity of water, only negative results were obtained. A thorium salt added to sea-water gives an immediate precipitate (which is cleared by an acid). It may be that the poverty in thorium of the oceanic waters and the underlying sediments is not a true indication of the thorium content of the rocks, but is to be referred to chemical effects consigning the thorium to the littoral deposits.

The geological bearing of the distribution of thorium cannot be discussed without a knowledge of the rate of transformation of that element. According to Bragg (Phil. Mag. June 1906) this element is breaking up at one-fifth the rate of uranium. If this is so, and taking into account the lesser emission of α particles by the derivatives of thorium, the indications of the results given above assign to thorium a relatively unimportant position compared with uranium as a heat-producing agent in the rocks. On the other hand, if the rates of transformation are about equal (Soddy, Phil. Mag. Oct. 1908), the importance of thorium might be comparable with that of uranium, the quantities of the two elements in the rocks not appearing to differ greatly in amount.

LXXI. *On Induced Stability.* *By* ANDREW STEPHENSON *.

1. NOT only may a single pivoted body be maintained in the position of unstable equilibrium by vertical oscillation of the pivot, but furthermore a number of freely jointed links may be similarly maintained.

In the simplest case, when there are only two links, the conditions of stability are readily found. With the usual notation the equations of motion in the neighbourhood of the vertical are

$$\{(h_1^2+k_1^2+\mu l_1^2)D^2-(g+2\alpha n^2\cos nt)(h_1+\mu l_1)\}\theta+\mu l_1h_2D^2\phi=0,$$
$$\{(h_2^2+k_2^2)D^2-(g+2\alpha n^2\cos nt)h_2\}\phi+l_1h_2D^2\theta=0,$$

where μ is the ratio of the mass of the second, or upper, rod

* An addition to a paper under the above title, Feb. 1908. Communicated by the Author.

Маятник Капицы

История вопроса

1. Капица П.Л. “Динамическая устойчивость маятника при колеблющейся точке подвеса” *ЖЭТФ* **21** 588 – 597 (1951)

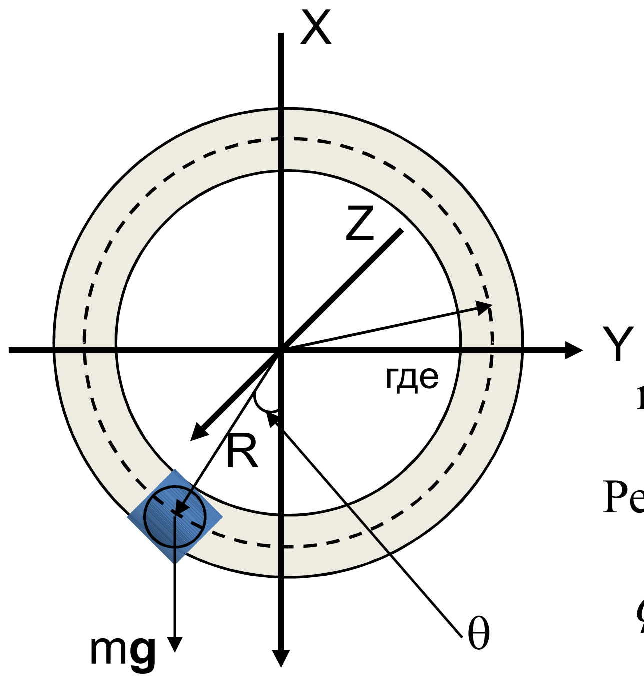
2. Капица П.Л. “Маятник с вибрирующим подвесом” *УФН* **44** 7 – 20 (1951)

Идея выделения в движении маятника быстро и медленно меняющихся составляющих
1934, 1937

Принцип жесткой фокусировки заряженных частиц
и маятник Капицы
1953

Маятник Капицы

Критерии



x, y – координаты центра шарика

$$x^2 + y^2 = R^2$$

$X_т(t) = A\cos(\Omega t)$ – движение трубки по вертикали

Уравнение моментов:

$$mR^2\ddot{\theta}(t) = -mgR\cdot\sin\theta(t) - mA\Omega^2\cdot\cos(\Omega t)\cdot R\sin\theta(t)$$

Решение уравнения представим в виде: $\theta(t) = \phi(t) + \alpha(t)$

$\phi(t)$ – «медленная» и $\alpha(t)$ – «быстрая» составляющие

$\alpha(t) = \frac{A}{R}\cdot\cos(\Omega t)\cdot\sin\phi(t)$ Тогда $\ddot{\alpha}(t) \approx -\frac{A}{R}\Omega^2\cdot\cos(\Omega t)\cdot\sin\phi(t)$

Учитывая малость отношения $\frac{A}{R}$ после преобразований получим

$$\ddot{\phi}(t) + \left[\frac{A^2}{2R^2}\Omega^2\cdot\cos\phi(t) + \frac{g}{R}\right]\sin\phi(t) = 0$$

Маятник Капицы

Критерии

Если теперь определить стандартным образом эффективную потенциальную энергию U_{eff}, отвечающую величине $\phi(t)$ при помощи соотношения $\ddot{\phi}(t) = -\frac{d}{d\phi}U_{eff}$, то

$$U_{eff} = -\frac{\mathrm{g}}{\mathrm{R}}\cdot\cos\phi + \frac{1}{4}\left(\frac{\mathrm{A}}{\mathrm{R}}\Omega\cdot\sin\phi\right)^2$$

Откуда получим три равновесных положения системы:

$$\phi_1 = 0, \phi_2 = \pi, \cos\phi_3 = -\frac{2\mathrm{gR}}{\Omega^2\mathrm{A}^2}$$

$\phi_1 = 0$ устойчиво всегда

$\phi_2 = \pi$ будет устойчивым только при выполнении неравенства $\boxed{\frac{2\mathrm{gR}}{\Omega^2\mathrm{A}^2} \le 1}$

ϕ_3 всегда неустойчиво

Уточненный критерий: $\boxed{(\Omega\mathrm{A})^2 \ge 2\mathrm{gL}\left(1 + \frac{I}{mL^2}\right)}$